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A note on the non-symmetric equations of connection in Einstein's unified field theory.

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Consider a pair of varieties V_1 and V_2 embedded in a Minkowskian n-space M_n At corresponding points, P, Q of the varieties V_1, V_2 let us denote the two sets of tangent vectors by X_μ, Y_ν, the associated contravariant vectors being represented by X^μ, Y^ν A non-symmetric tensor $g_{\mu\nu}$ with 16 components is now defined by the scalar products $\left\{\begin{matrix} X_\mu \\ Y_\nu \end{matrix}\right\}$ of the tangent vectors X_μ, Y_ν With no restrictions imposed on the varieties it has been shown (Ghosh 1973) that the symmetric part of $g_{\mu\nu}$ represents the gravitational tensor $S_{\mu\nu}$ and the anti-symmetric part the electromagnetic tensor $F_{\mu\nu}$ To derive the system of non-symmetric equations of connection we introduce sets of modified tangent vectors $X^{\hat{\mu}}$, $Y^{\hat{\mu}}{}_\delta$, $X_{\hat{\lambda}}$. $Y_{\hat{\lambda}}$ defined by

$$X^{\hat{\mu}} = X_\lambda \left\{\begin{matrix} X^\lambda \\ Y^\mu \end{matrix}\right\}, Y^{\hat{\mu}} = Y_\lambda \left\{\begin{matrix} Y^\lambda \\ X^\mu \end{matrix}\right\} X_{\hat{\lambda}} = X^\mu \left\{\begin{matrix} X_\mu \\ Y_\lambda \end{matrix}\right\} Y_{\hat{\lambda}} = Y^{\mu\delta} \left\{\begin{matrix} Y_\mu \\ X_\lambda \end{matrix}\right\} \quad .. \quad (1)$$

where $\left\{\begin{matrix} X_\mu \\ Y_\lambda \end{matrix}\right\}$ denotes $g_{\mu\lambda}$ and $\left\{\begin{matrix} X^\lambda \\ Y_\mu \end{matrix}\right\}$ denotes $g^{\mu\lambda}$

From eq. (1) one can show

$$\left\{\begin{matrix} X^{\hat{\mu}} \\ Y_{\hat{\lambda}} \end{matrix}\right\} = \delta_\lambda{}^\mu = \left\{\begin{matrix} Y^{\hat{\mu}} \\ X_{\hat{\lambda}} \end{matrix}\right\} \quad \ldots \quad (2)$$

We take note of the following relations among hooked vectors

$$X^{\hat{\mu}} = X_{\hat{\lambda}} \left\{\begin{matrix} Y^{\hat{\lambda}} \\ X^{\hat{\mu}} \end{matrix}\right\}, Y^{\hat{\mu}} = Y_{\hat{\lambda}} \left\{\begin{matrix} X^{\hat{\lambda}} \\ Y^{\hat{\mu}} \end{matrix}\right\}, X_{\hat{\lambda}} = X^{\hat{\mu}} \left\{\begin{matrix} Y_{\hat{\mu}} \\ X_{\hat{\lambda}} \end{matrix}\right\} Y_{\hat{\lambda}} = Y^{\hat{\mu}} \left\{\begin{matrix} X_{\hat{\mu}} \\ Y_{\hat{\lambda}} \end{matrix}\right\} \quad \ldots \quad (3)$$

Consider now the identical relation

$$\delta_\sigma \left\{\begin{matrix} X_{\hat{\mu}} \\ Y_{\hat{\nu}} \end{matrix}\right\} = \left\{\begin{matrix} X_{\hat{\mu}} \\ Y_{\hat{\lambda}} \end{matrix}\right\} \left\{\begin{matrix} X^{\hat{\lambda}} \\ Y_{\hat{\nu}\sigma} \end{matrix}\right\} + \left\{\begin{matrix} X_{\hat{\lambda}} \\ Y_{\hat{\nu}} \end{matrix}\right\} \left\{\begin{matrix} Y^{\hat{\lambda}} \\ X_{\hat{\mu}\sigma} \end{matrix}\right\} \quad .. \quad (4)$$

If we postulate

$$\left\{\begin{matrix} X^{\hat{\lambda}} \\ Y_{\hat{\nu}\sigma} \end{matrix}\right\} = \left\{\begin{matrix} Y^{\hat{\lambda}} \\ X_{\hat{\sigma}\nu} \end{matrix}\right\} \quad \ldots \quad (5)$$

the above yields the system of 64 equations of connection in Einstein's unified field theory

We now adopt Bose's method of reduction of the system of eqs. (4) (Bose 1954)

Combining eq (4) with the conjugate (X, Y interchanged) we get

$$\partial_\sigma \hat{S}_{\mu\nu} = \hat{S}_{\rho\nu}\hat{P}^{\rho}{}_{\mu\sigma} - \hat{F}_{\rho\nu}\hat{V}^{\rho}{}_{\rho\mu} + \hat{S}_{\rho\mu}P^{\rho}{}_{\nu\sigma} - \hat{F}_{\rho\mu}\hat{V}^{\rho}{}_{\nu\sigma}, \quad \dots \quad (6)$$

$$\delta_\sigma \hat{F}_{\mu\nu} = -\hat{S}_{\rho\nu}\hat{V}^{\rho}{}_{\mu\sigma} + \hat{F}_{\rho\nu}\hat{P}^{\rho}{}_{\mu\sigma} + \hat{S}_{\rho\mu}\hat{V}^{\rho}{}_{\nu\sigma} - \hat{F}_{\rho\mu}\hat{P}^{\rho}{}_{\nu\sigma}, \quad \dots \quad (7)$$

where

$$\hat{S}_{\mu\nu} = \tfrac{1}{2}\left(\left\{ \begin{matrix} X'\hat{\mu} \\ Y\hat{\nu} \end{matrix} \right\} + \left\{ \begin{matrix} Y\hat{\mu} \\ X\hat{\nu} \end{matrix} \right\}\right), \quad \hat{F}_{\mu\nu} = \tfrac{1}{2}\left(\left\{ \begin{matrix} X\hat{\mu} \\ Y\hat{\nu} \end{matrix} \right\} - \left\{ \begin{matrix} Y\hat{\mu} \\ X\hat{\nu} \end{matrix} \right\}\right)$$

$$\hat{P}^{\rho}{}_{\mu\sigma} = \tfrac{1}{2}\left(\left\{ \begin{matrix} X^{\hat{\rho}} \\ Y_{\hat{\mu}\sigma} \end{matrix} \right\} + \left\{ \begin{matrix} Y^{\hat{\rho}} \\ X_{\hat{\mu}\sigma} \end{matrix} \right\}\right), \quad \hat{V}^{\rho}{}_{\mu\sigma} = \tfrac{1}{2}\left(\left\{ \begin{matrix} X^{\hat{\rho}} \\ Y_{\hat{\mu}\sigma} \end{matrix} \right\} - \left\{ \begin{matrix} Y^{\hat{\rho}} \\ X_{\hat{\mu}\sigma} \end{matrix} \right\}\right)$$

By virtue of relations (5) we notice

$$\hat{P}^{\rho}{}_{\mu\sigma} = \hat{P}^{\rho}{}_{\sigma\mu} \quad \hat{V}^{\rho}{}_{\mu\sigma} = -V^{\hat{\rho}}{}_{\sigma\mu}$$

On putting

$$\hat{S}_{\rho\nu}\hat{P}^{\rho}{}_{\mu\sigma} = P_{\nu\mu\sigma}, \; \hat{S}_{\rho\nu}\hat{V}^{\rho}{}_{\mu\sigma} = T_{\nu\mu\sigma}, \; \hat{S}^{\rho\lambda}\hat{F}_{\rho\mu} = C^{\lambda}{}_{\mu}$$

Eqs (6) and (7) reduce to the forms

$$\partial_\sigma \hat{S}_{\mu\nu} = P_{\nu\mu\sigma} - C^{\lambda}{}_{\nu}T_{\lambda\mu\sigma} + P_{\mu\nu\sigma} - C_{\mu}{}^{\lambda}T_{\lambda\nu\sigma}, \quad \dots \quad (8)$$

$$\partial_\sigma \hat{F}_{\mu\nu} = -T_{\nu\mu\sigma} + C_{\nu}{}^{\lambda}P_{\lambda\mu\sigma} + T_{\mu\nu\sigma} - C_{\mu}{}^{\lambda}P_{\lambda\nu\sigma} \quad \dots \quad (9)$$

From eq (8) one can show

$$\tfrac{1}{2}(\partial_\mu \hat{S}_{\nu\sigma} + \partial_\nu \hat{S}_{\mu\sigma} - \partial_\sigma \hat{S}_{\mu\nu}) = P_{\sigma\mu\nu} + C_{\mu}{}^{\lambda}T_{\lambda\sigma\sigma} + C_{\nu}{}^{\lambda}T_{\lambda\mu\sigma} \quad \dots \quad (10)$$

Using eq. (10) in eq (8) one can easily identify $\hat{S}_{\mu\nu}$ with the gravitational tensor.

From (9) again, we have

$$\tfrac{1}{2}(\partial_\sigma \hat{F}_{\mu\nu} + \partial_\mu F_{\nu\sigma} + \partial_\nu \hat{F}_{\sigma\mu}) = T_{\sigma\mu\nu} + T_{\mu\nu\sigma} + T_{\nu\sigma\mu} \quad \dots \quad (11)$$

This shows that $\hat{F}_{\mu\nu}$ can be identified with the electromagnetic tensor if the hooked field satisfies the equation

$$T_{\sigma\mu\nu} + T_{\mu\nu\sigma} + T_{\nu\sigma\mu} = 0$$

References

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